

Nambu-Jona-Lasinio Model at the Next-to-Leading Order in $1/N$ ¹K. Akama^a^aDepartment of Physics, Saitama Medical College, Kawakado, Moroyama, Saitama 350-04, Japan

We derived and solved the compositeness condition in the Nambu-Jona-Lasinio model at the next-to-leading order in $1/N$, and obtained the expressions for the effective coupling constants in terms of the compositeness scale. In the NJL model with a scalar composite, the next to leading contributions are too large for $N = 3$. In the induced gauge theory with abelian gauge symmetry, the correction term is reasonably suppressed, while, in the $SU(N_c)$ gauge theory with N_f flavors of fermions, the correction is suppressed only for $N_f > 11N_c/2$, complementarily to asymptotic freedom.

1. INTRODUCTION

This talk is based on the works done in collaboration with Takashi Hattori from Kanagawa Dental College [1]. The Nambu-Jona-Lasinio model is a model of fermions interacting through a chirally symmetric four-fermion interaction, through which composite bosons are formed [2]. This model offers us a simple field theoretical scheme which realizes tractable compositeness and spontaneously broken symmetry [3]. It has, however, theoretical drawbacks due to its non-renormalizability. Then the following questions are frequently asked: “How can we regulate the badly divergent higher order contributions?” “Can we believe the lowest order results in spite of the badly divergent contributions?” “How should we take into account the quantum effects of the composite bosons?” The purpose of this talk is to answer these questions from the view points of the $1/N$ expansion [4], where N is the number of the fermion species. We first note that the Nambu-Jona-Lasinio model is a special case of the renormalized Yukawa model under the compositeness condition $Z_3 = 0$ [5], where Z_3 is the wave-function renormalization constant of the to-be-composite boson. Then, we can calculate everything in the Nambu-Jona-Lasinio model by calculating the corresponding quantity in the well-understood renormalized Yukawa model, and after that by imposing the compositeness condition. Here we work out the compositeness condition at the next-to-leading order in $1/N$, and solve it to obtain the coupling constant in terms of the compositeness scale.

2. NAMBU-JONA-LASINIO MODEL

We consider the NJL model for the fermion $\psi = \{\psi_1, \psi_2, \dots, \psi_N\}$ with N colors interacting through the four fermion interaction with $U(1) \times U(1)$ chiral symmetry:

$$\mathcal{L}_{\text{NJL}} = \bar{\psi} i \not{\partial} \psi + f |\bar{\psi}_L \psi_R|^2. \quad (1)$$

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In 3+1 dimensions, it is not renormalizable, and we assume a very large but finite momentum cutoff which is taken as the compositeness scale. This Lagrangian \mathcal{L}_{NJL} is known to be equivalent to the linearized Lagrangian [6]

$$\mathcal{L}'_{\text{NJL}} = \bar{\psi} i \not{\partial} \psi + (\bar{\psi}_L \phi \psi_R + \text{h.c.}) - \frac{1}{f} |\phi|^2 \quad (2)$$

which is written in terms of the auxiliary field ϕ . Now compare this with the renormalized Yukawa model

$$\mathcal{L}_{\text{Yukawa}} = Z_\psi \bar{\psi}_r i \not{\partial} \psi_r + Z_g g_r (\bar{\psi}_{rL} \phi_r \psi_{rR} + \text{h.c.}) + Z_\phi |\partial_\mu \phi_r|^2 - Z_\mu \mu_r^2 |\phi_r|^2 - Z_\lambda \lambda_r |\phi_r|^4 \quad (3)$$

where the quantities with the index r are the renormalized ones, g and λ are the effective coupling constants, μ is the boson mass, and Z 's are the renormalization constants. We can see that they share the fermion kinetic term, fermion-boson interaction term and the boson mass term, but the boson kinetic term and the four boson interaction term are absent in the NJL Lagrangian. Then, if

$$Z_\phi = Z_\lambda = 0, \quad (4)$$

the Yukawa model Lagrangians reduces to the NJL model Lagrangian, as far as we identify

$$\psi = \sqrt{Z_\psi} \psi_r, \quad \phi = \frac{Z_g}{Z_\psi} g_r \phi_r, \quad f = \frac{Z_g^2 g_r^2}{Z_\psi^2 Z_\mu \mu_r^2} \quad (5)$$

in the Yukawa model. The condition (4) is called the compositeness condition [5]. Thus NJL model is the special case of the well-understood renormalized Yukawa model specified by the compositeness condition. The compositeness condition gives rise to relations among coupling constants g_r , λ_r , and the cut off Λ . If the chiral symmetry is spontaneously broken, they imply relations among the fermion mass m_f , the Higgs-scalar mass M_H , and the cutoff Λ . Thus we can analyze everything in the NJL model by investigating the well-understood Yukawa model, and by imposing the compositeness condition on the coupling constants and masses. Then what is urgent is to work out the compositeness condition, and solve it for the coupling constants.

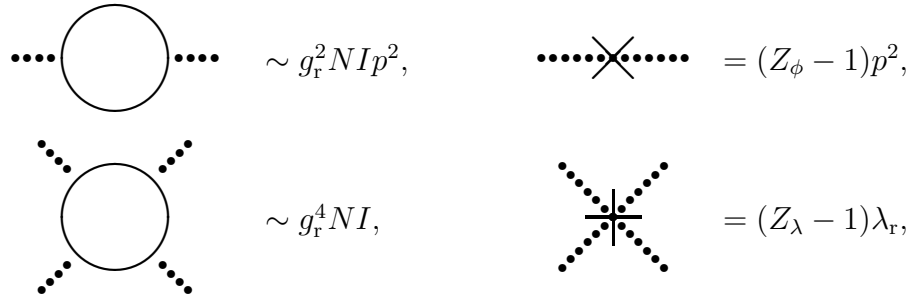


Fig. 1 The boson self-energy part and the four-boson vertex part in the lowest order in $1/N$. The lines ——— and stand for the fermion and boson propagator, respectively.

For an illustration, we begin with the lowest-order contributions in $1/N$ expansion. In the Yukawa model, the boson self-energy part and the four-boson vertex part are given by the diagrams in Fig. 1 and their counter terms, where I is the divergent integral:

$$I = \begin{cases} \frac{1}{16\pi^2} \frac{1}{\epsilon} & \text{(dimensional regularization) } \left(\epsilon = \frac{4-d}{2} \right) \\ \frac{1}{16\pi^2} \log \Lambda^2 & \text{(Pauli Villars regularization)} \end{cases} \quad (6)$$

The renormalization constants Z_ϕ and Z_λ should be chosen as

$$Z_\phi = 1 - g_r^2 NI, \quad Z_\lambda \lambda_r = \lambda_r - g_r^4 NI \quad (7)$$

so as to cancel out all the divergences in Fig. 1. Then the compositeness condition is obtained by simply putting Z_ϕ and Z_λ vanishing,

$$0 = 1 - g_r^2 NI, \quad 0 = \lambda_r - g_r^4 NI, \quad (8)$$

and it is easily solved to give the expressions for the coupling constants [7].

$$g_r^2 = \frac{1}{NI}, \quad \lambda_r = \frac{1}{NI}. \quad (9)$$

If $\mu_r < 0$, the chiral symmetry is spontaneously broken, and the masses of the physical fermion and physical Higgs scalar are given in terms of cutoff:

$$m_f = g_r \langle \phi \rangle = \langle \phi \rangle / \sqrt{NI}, \quad M_H = 2\sqrt{\lambda_r} \langle \phi \rangle = 2\langle \phi \rangle / \sqrt{NI} \quad (10)$$

The Higgs mass is twice the fermion mass.

$$2m_f = M_H \quad (11)$$

These reproduce the well known results of the lowest order Nambu-Jona-Lasinio model [2].

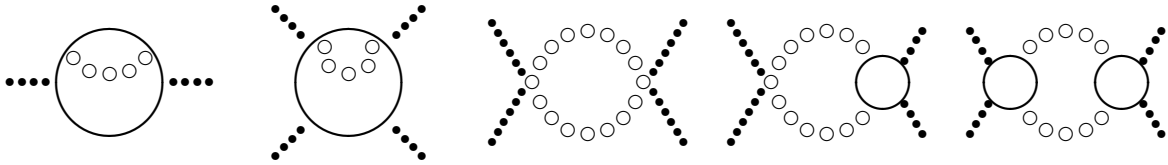


Fig. 2 The boson self-energy part and the four-boson vertex part in the next-to-leading order in $1/N$. The chains of tiny circles stand for the infinite sum of the fermion loop insertions into the boson propagator.

Now we turn to the next-to-leading order in $1/N$ expansion. In the Yukawa model, the boson self-energy part and the four boson vertex part are given by the diagrams in Fig. 2 and the counter terms for all the subdiagram divergences. The renormalization constants Z_ϕ and Z_λ are calculated to be like

$$Z_\phi = 1 - g_r^2 NI - g_r^2 I - \frac{1}{N} (1 - g_r^2 NI) \log(1 - g_r^2 NI) \quad (12)$$

$$\begin{aligned}
Z_\lambda \lambda_r = & \lambda_r - g_r^4 N I + 8g_r^4 I + \frac{20(\lambda_r - g_r^2)^2 I}{1 - g_r^2 N I} \\
& - \frac{1}{N} \left[2g_r^2(1 - g_r^2 N I) + 20(\lambda_r - g_r^2) \right] \log(1 - g_r^2 N I)
\end{aligned} \tag{13}$$

so as to cancel out all the divergences in Fig. 2. The logarithm arises from the infinite sum over the fermion loop insertions into the internal boson lines. The compositeness condition is given by putting these expressions vanishing. Though it looks somewhat complicated at first sight, it can be solved by iteration to give this very simple solution.

$$g_r^2 = \frac{1}{NI} \left[1 - \frac{1}{N} + O\left(\frac{1}{N^2}\right) \right], \quad \lambda_r = \frac{1}{NI} \left[1 - \frac{10}{N} + O\left(\frac{1}{N^2}\right) \right]. \tag{14}$$

The next-to-leading correction to the ratio of M_H and m_f , which was 2 in the lowest order, is calculated to be:

$$\frac{M_H}{m_f} = \frac{g_r}{\sqrt{\lambda_r}} = 2 \left[1 - \frac{9}{2N} + O\left(\frac{1}{N^2}\right) \right] \tag{15}$$

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For the case of $N = 3$ of the practical interest, the corrections turn out to be too large, and the coupling constant λ is negative, which implies that the Higgs potential is unstable. This seems to be a serious difficulty of this model. A way out of this difficulty is as follows. In practical applications, ψ is the quark and ϕ is the meson. Then it is rather natural to take the cutoff Λ_ψ for the quark propagator is much larger than the cutoff Λ_ϕ for the meson propagator. An actual calculation shows that the NLO correction terms are suppressed by a factor

$$r = \log \Lambda_\phi^2 / \log \Lambda_\psi^2, \tag{16}$$

and the higher order terms in $1/N$ are further suppressed.

3. INDUCED GAUGE THEORY

We can apply [1] this method to the induced gauge theory [8], namely, the gauge theory with a composite gauge field. It is given by the strong coupling limit $f \rightarrow \infty$ [9] of the vector-type four Fermi interaction model for the fermion ψ with the mass m :

$$\mathcal{L}_{4F} = \bar{\psi}_j (i\partial - m) \psi - f (\bar{\psi} \gamma_\mu \psi)^2, \tag{17}$$

where f is the coupling constant. The Lagrangian \mathcal{L}_{4F} is equivalent to

$$\mathcal{L}'_{4F} = \bar{\psi} (i\partial - m - A) \psi \tag{18}$$

written in terms of the vectorial auxiliary field A_μ . Then we can see that this is the special case of the renormalized gauge theory

$$\mathcal{L}_G = \bar{\psi}_r (iZ_2\partial - Z_m m_r - Z_1 e_r A_r) \psi_r - \frac{1}{4} Z_3 (\partial_\mu A_{r\nu} - \partial_\nu A_{r\mu})^2, \tag{19}$$

specified by the compositeness condition

$$Z_3 = 0, \quad (20)$$

where the quantities indicated by suffices r are renormalized ones, e is the effective coupling constant, and Z 's are the renormalization constants.

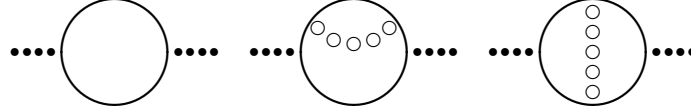


Fig. 3 The gauge boson self-energy part in the lowest and next-to-leading order in $1/N$.

The self-energy part of the gauge boson at the leading and the next-to-leading order is given by the diagrams in Fig. 3. The renormalization constant Z_3 is chosen so as to cancel out the divergences in these diagrams. After a lengthy calculation we obtain the following expression for Z_3 :

$$Z_3 = 1 - \frac{e_r^2 N}{12\pi^2 \epsilon} - \frac{3e_r^2}{16\pi^2} \left[1 + \left(1 - \frac{12\pi^2 \epsilon}{e_r^2 N} \right) \ln \left(1 - \frac{e_r^2 N}{12\pi^2 \epsilon} \right) \right], \quad (21)$$

where $\epsilon = (4 - d)/2$ with the dimension d . Then the compositeness condition $Z_3 = 0$ is solved to give the simple solution:

$$e_r^2 = \frac{12\pi^2 \epsilon}{N} \left[1 - \frac{9\epsilon}{4N} \right]. \quad (22)$$

The correction term $9\epsilon/4N$ is naturally suppressed by the small factor ϵ . It justifies the lowest order approximation of this model unlike in the case of the aforementioned NJL model of the scalar composite. The origin of the suppression factor is traced back to the gauge cancellation of the leading divergence in the next-to-leading (in $1/N$) diagrams in Fig. 3.

So far we assumed that all the fermions has the same charges for simplicity. If the charges Q_i are different, the expression is modified as

$$e_r^2 = \frac{12\pi^2 \epsilon}{\sum_j Q_j^2} \left[1 - \frac{9\epsilon \sum_j Q_j^4}{4(\sum_j Q_j^2)^2} \right]. \quad (23)$$

If we apply this to the quantum electrodynamics with 3 generations of quarks and leptons, ϵ is estimated to be 6×10^{-3} , which implies the next-to-leading order correction amounts only to 0.1% of the lowest order term.

Now we apply it [1] to the non-abelian induced gauge theory. We consider the system of N_c gauged colors and N_f ungauged flavors. The starting four Fermi interaction

$$\mathcal{L}_{4F} = \bar{\psi} (i\partial - m) \psi - f \left(\bar{\psi} \lambda^a \gamma_\mu \psi \right)^2, \quad (24)$$

involves the internal symmetry expressed by the Gell-Mann matrix $\lambda^a (a = 1, \dots, N_c^2 - 1)$, and the auxiliary field A_μ^a in the equivalent Lagrangian (in the limit $f \rightarrow \infty$)

$$\mathcal{L}'_{4F} = \bar{\psi} (i\partial - m - \lambda_a A^a) \psi \quad (25)$$

also has an internal symmetry. Then this is the special case of the renormalized gauge theory

$$\begin{aligned} \mathcal{L}_G = & \bar{\psi}_r \left(iZ_2 \not{\partial} - Z_m m_r - Z_1 g_r \lambda^a A_r^a \right) \psi_r \\ & - \frac{1}{4} Z_3 \left(\partial_\mu A_{r\nu}^a - \partial_\nu A_{r\mu}^a + Z_3^{1/2} Z_g g_r f^{abc} A_{r\mu}^b A_{r\nu}^c \right)^2, \end{aligned} \quad (26)$$

specified by the compositeness condition

$$Z_3 = 0, \quad (27)$$

where the quantities indicated by suffices r are renormalized ones, g is the effective coupling constant, and Z 's are the renormalization constants.

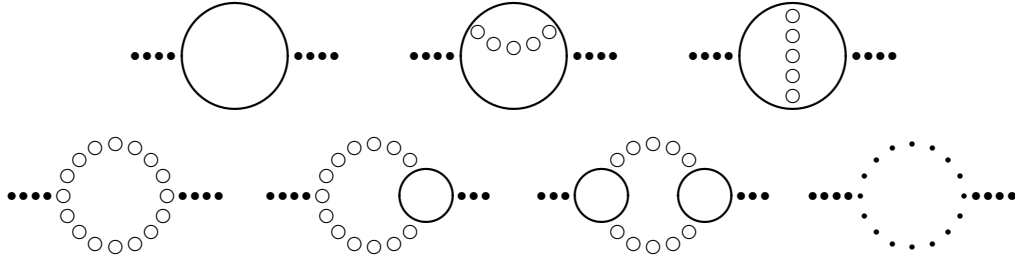


Fig. 4 The gauge boson self-energy part in the lowest order and next-to-leading order in $1/N$. The line \cdots stands for the Fadeev Popov ghost propagator.

The self-energy part of the gauge boson is given by the diagrams in Fig. 4 at the leading order and the next-to-leading order in $1/N_f$. The renormalization constant Z_3 is chosen so as to cancel out the divergences in these diagrams. After a lengthy calculation we obtain the following expression.

$$\begin{aligned} Z_3 = & 1 - \frac{2}{3} N_f g_r^2 I + \frac{11}{3} N_c g_r^2 I - \frac{\alpha}{2} N_c g_r^2 I \left(1 - \frac{2}{3} N_f g_r^2 I \right) \\ & + \frac{3}{2} N_c \left(\frac{3}{2N_f} - g_r^2 I \right) \ln \left(1 - \frac{2}{3} N_f g_r^2 I \right) + O\left(\frac{1}{N_f^3}\right). \end{aligned} \quad (28)$$

Then the compositeness condition $Z_3 = 0$ is solved to give the simple solution:

$$g_r^2 = \frac{3}{2N_f I} \left[1 + \frac{11N_c}{2N_f} + O\left(\frac{1}{N_f^2}\right) \right]. \quad (29)$$

Though the Z_3 itself does depend on gauge parameter α , the solution g_r to the compositeness condition $Z_3 = 0$ does not. This should be so because the coupling constants and the compositeness scale are observable objects. The solution is true only when the next-to-leading contribution is smaller than the leading order one. This implies the relation

$$N_f > \frac{11N_c}{2}. \quad (30)$$

Note that this region for N_f and N_c is complementary to that for asymptotic freedom. When the gauge theory is asymptotically free, the next-to-leading contributions are too

large, so that the gauge bosons cannot be a composite of the above type. And when it is asymptotically non-free, the next-to-leading order contributions are reasonably suppressed, and the gauge boson can be interpreted as a composite.

4. SUMMARY

In summary, The composite dynamics of the NJL model is clarified by analyzing the well-understood renormalized Yukawa model with the compositeness condition. We derived and solved the compositeness condition in various models at the next-to-leading order, and obtained the expressions for the effective coupling constants in terms of the compositeness scale. In the NJL model with a scalar composite, the corrections are $-1/N$ and $-10/N$, which are too large for $N = 3$ of our practical interest, and furthermore λ is negative, which implies an unstable Higgs potential. A way out of this difficulty is to use the fact that the cutoff for the composite boson is much smaller than the cutoff for the elementary fermion. On the other hand, in the induced gauge theory with abelian gauge symmetry, the correction term is $-9\epsilon/4N$, which is reasonably suppressed. In the non-abelian gauge theory, the correction is $11N_c/2N_f$. This is true only when the next-to-leading contribution is smaller than the leading order one, which implies that $N_f > 11N_c/2$. This condition for N_f and N_c is complementary to that of asymptotic freedom. We expect that the methods and results presented here will be useful in pursuing composite dynamics of nuclei and hadrons, as well as possible compositeness of gauge bosons.

REFERENCES

1. K. Akama, Phys. Rev. Lett. **76** (1996) 184; K. Akama and T. Hattori, Phys. Lett. **B392** (1997) 383. K. Akama and T. Hattori, in preparation.
2. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345.
3. See for example, K. Akama and H. Terazawa, INS-Rep-**257** (1976); H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. **D15** (1977) 480.
4. J. Zinn-Justin, Nucl. Phys. **B367**, 105 (1991); D. Lurie and G.B. Tupper, Phys. Rev. **D47** (1993) 3580; J. A. Gracey, Phys. Lett. **B308**, 65 (1993); **B342**, 297 (1995). E. Pallante, NORDITA preprint, NORDITA 96/31 N, P (1996), hep-ph/9606286; G. Cvetic and N. D. Vlachos, Phys. Lett. **B377**, 102 (1996); R. de Mello Koch and J. P. Roderigues, Phys. Rev. **D54**, 7794 (1996). For further references, see Ref. [1]
5. B. Jovet, Nuovo Cim. **5** (1956) 1133; M. T. Vaughn, R. Aaron and R. D. Amado, Phys. Rev. **124** (1961) 1258; A. Salam, Nuovo Cim. **25** (1962) 224; S. Weinberg, Phys. Rev. **130** (1963) 776.
6. T. Kugo, Prog. Theor. Phys. **55** (1976) 2032; T. Kikkawa, Prog. Theor. Phys. **56** (1976) 947.
7. T. Eguchi, Phys. Rev. **D14** (1976) 2755; **D17** (1978) 611; D. Campbell, F. Cooper, G. S. Guralnik and N. Snyderman, Phys. Rev. **D19** (1979) 549; K. Shizuya, Phys. Rev. **D21**, 2327 (1980).
8. W. Heisenberg, Rev. Mod. Phys. **29** (1957) 269; J. D. Bjorken, Ann. Phys. **24** (1963) 174. For further references, see Akama and Hattori [1].
9. I. Bialynicki-Birula, Phys. Rev. **130** (1963) 465.